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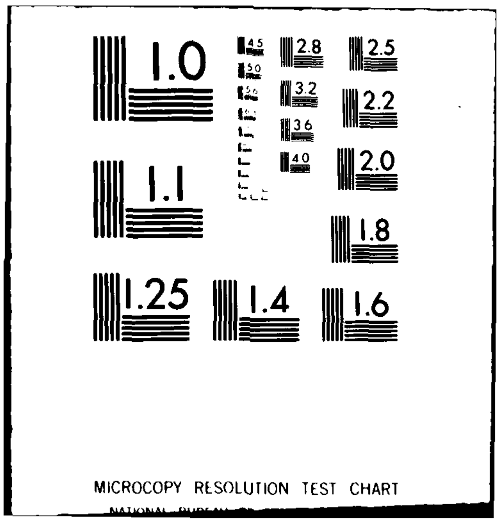
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This paper examines anomalous transport in a reversed field theta pinch. The principal effects are anomalous resistivity and rotation generation. Similarity solutions for the resistive decay are found which agree qualitatively with experiment. Also it is shown that the spin up and anomalous resistivity may be the effect of a single underlying cause, a current driven microinstability.

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CONTENTS

TRANSPORT STUDIES IN REVERSED FIELD THETA PINCHES

I. INTRODUCTION

This paper studies transport processes in reversed field theta pinches (RFPs). In such a pinch an initial bias field points in the negative z direction. Then the main θ pinch coil, which produces a positive B_z is pulsed on. The idea is that the negative and positive magnetic fields reconnect, either spontaneously or with the aid of forcing coils, to form a closed field region as shown in Figure 1. Just outside this closed field region are open field lines which intersect the wall. A plasma which is trapped in the region of closed field lines can be effectively isolated from all external surroundings. Recent experiments on the FRX devices at Los Alamos,[1] as well as earlier experiments on Pharos at the Naval Research Laboratory[2] and also other experiments[3] show that these configurations can be produced and apparently are stable to magnetohydrodynamic modes (i.e., they live for many tens, or hundreds of Alfv̀en times). Also, they are quite elongated, which indicates that there is a great deal to learn with a one dimensional theory.

If RFPs are stable to gross magnetohydrodynamic modes, the next problem is the effect of transport. This paper is specifically motivated by the Los Alamos experiments, although these transport effects have also been seen in nearly all other reversed field θ pinch experiments. There are two striking transport effects in FRX.[1] First of all, there is the decay of the plasma, apparently via resistive diffusion, although the decay rate is much higher than one would expect on the basis of classical

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collisions. Secondly there is the generation of rotation in the plasma. As the plasma decays, it spins up to about the ion diamagnetic velocity, at which point it disrupts via the onset of an $m = 2$ rotational instability. A significant experimental fact is that the plasma spins up to the ion diamagnetic velocity in about the resistive decay time, as measured by plasma loss. This fact motivates our hypothesis that there is a single underlying mechanism which is responsible for both the decay and spin-up.

Other theoretical work on the decay was done by Hamasaki and Linford,[4] who used a one dimensional, high β transport code to study the decay of a reversed field θ-pinch plasma. There are several theories of the spin-up. Steinhauer[5] invokes end shorting so that $E_r = 0$ for the plasma on the open field lines. This plasma then rotates at the ion diamagnetic frequency, and the rotation is transported inward via classical shear viscosity. In order to use this theory, of course, the rotational velocity in the sheath plasma, which probably has gradient scale lengths of an ion cyclotron frequency or less, as well as high speed convection along the field lines, must be accurately known. Barnes and Seyler[6] explain rotation by examining the angular momentum of a particle just as it crosses the separatrix. They find that such a particle does have a particular angular momentum, so that as particles diffuse across the separatrix, the remaining plasma spins up in the opposite direction. This theory is like ours in that it relates the spin up to the particle loss. Fang and Miley[7] also examine the effect of particle transport on rotation.

Our work begins in Section II by reviewing the experimental data for FRX.[1] We conclude that the resitivity is almost certainly anomalous by at least two orders of magnitude. The viscosity also seems to be anomalous by more than one order of magnitude, unless the rotation speed on the open field lines is of order of the ion thermal speed. The thermal conduction losses however, are small compared to diffusive losses. Thermal conduction therefore does not seem to be an important effect, and it may well be classical. Another striking feature of the decay is that the separatrix stays nearly fixed in time and the density profile decays, but does not change its structure very much. This fact leads us to attempt a similarity solution for the resistive decay of a reversed field θ pinch.

Section III shows that similarity solutions for the decay can be found. While the problem we solve is somewhat idealized, solutions can be found much more simply and economically than with a transport code, and a great deal of useful information can be obtained. For instance we show that the resistive decay of a reversed field θ pinch is quite different from either of the two separate decay processes which together constitute it; namely the pure resistive decay uncoupled to fluid motion, and the transport of plasma across a prescribed field, uncoupled to field decay. Probably the most interesting conclusion we arrive at is that the decay rate is quite insensitive to the boundary condition on the separatrix. We express this boundary condition as a ratio of density at the separatrix to that at the field null. Although the density and field structure, particularly near the

separatrix, does depend strongly on this ratio, the overall decay rate does not. Therefore our physical model does not require detailed knowledge of the behavior near the separatrix. Another interesting conclusion is that the decay rate remains finite as the resistivity at the neutral line goes to zero. This is the same conclusion obtained by Drake et al.,[8] using a quite different model for transport in a reversed field geometry.

Finally, Section IV describes our theory of rotation generation. It assumes that there is a current driven instability which gives rise to fluctuating fields in the plasma. The precise instability is not crucial to our model, but to be specific we consider a lower hybrid drift. This gives rise to an anomalous resitivity, which can be used in transport codes like that of Hamaskai.[4] However, one can show that it also gives rise to off diagonal terms[9] in the stress tensor, so that there is a net force on the plasma in the θ direction. In Section IV we calculate these off diagonal terms. While the spin up depends on the detailed nature of the turbulent spectrum, one could also calculate the ratio of spin up time to resistive diffusion time. This ratio turns out to be independent of spectrum and it depends only on the phase velocity of the fluctuating potential, the very quantity which is best known. It turns out that the plasma is indeed predicted to spin up to about the ion diamagnetic velocity in about a resistive diffusion time. Thus the anomalous resitivity and anomalous spin up appear to have a single underlying cause.

II. REVIEW OF FRX EXPERIMENTS

This section briefly reviews the information available from FRX experiments which has been reported at various meetings.[1] We do not discuss the circuitry required to set up the field reversed configuration, but discuss the decay of the plasma once it is initialized. At $t = 0$, the plasma is set up in a state like that shown in Figure 1. There is no poloidal field, and the plasma is quite elongated. The radius of the separatrix in FRX-B is about 5 or 6 centimeters, and the total length is about 80 centimeters. The electron temperature is typically 150 eV, nearly uniform in both space and time. The ion temperature and density are functions of fill pressure. As the fill pressure increases from 9 to 21 millitorr, the density increases from about 10^{15} cm^{-3} to 4×10^{15} cm^{-3}, and the ion temperature decreases from about 400 eV to about 100 eV.

Once the plasma is formed it decays over a time scale of about 40 μ sec. As it decays, it starts to rotate in the ion diamagnetic direction. The plasma finally disrupts because a rotational instability is excited when the rotation speed is of order the ion diamagnetic speed. It is also worth noting that at this point, roughly half of the plasma has been lost due to the slower decay. During this initial slow decay phase, the plasma basically retains its shape, both axially and radially. An estimate of the decay rate can be obtained from the decay in particle and energy inventory for the first 30 μ sec. The e-folding time for density is about 40 μ sec and the e-folding time for energy is slightly less. Since resistive (particle)

diffusion causes loss of both particles and energy, while thermal conduction causes loss of only energy, these results argue strongly that resistivity is more important than thermal conduction since the latter gives rise to the difference between the two decay rates. To summarize, the two most important transport effects occurring during the initial decay seem to be resistive diffusion and generation of rotation. Thermal conduction may also be playing a role, but it is probably a less important role.

We continue by examining qualitatively what classicial transport predicts for these effects. The resistive diffusion time is $\tau_r \approx 4\pi a^2/\eta c^2$ where a is roughly the separatrix radius and η is the resistivity, $\eta \approx 10^{-13}/T_e^{3/2}$. Taking a = 5 cm and T_e = 150 eV, we find that $\tau_r \approx 6 \times 10^{-3}$ sec. The observed decay time is more than two orders of magnitude smaller. Thus it seems certain that anomalous resistivity is playing an important role in FRX-B.

We next consider classical cross field ion thermal conduction. The thermal conduction coefficient is $K_\perp^i = 2nT_i/M_i^2\Omega_{ii}\tau_i$ where τ_i is the ion collision time, $\tau_i = 2 \times 10^6\ T_i/n$, assuming a value of 10 for the Coulomb logarithm. Making use of the fact that on the average, the beta is unity for a reversed field theta pinch configuration, we find that the thermal conduction time is given by

$\tau_c \approx 6 \times 10^{-9} \left(\frac{T_e+T_i}{T_i}\right) T_i^{3/2} a^2$, for T_i in eV and a in cm. Taking

T_i = 150 eV and a = 5 cm, we obtain $\tau_c \approx 1.4 \times 10^{-4}$ sec. This may be the explanation of the difference between the decay rates for energy and particle inventory. To confirm this, however, would require much more data than is currently available.

The other classical process is shear viscosity. The relevant ion viscosity coefficient is $\eta_1^i = \frac{3}{10} nT_i/\omega_{ii}^2\tau_i$. This has the same form as K_1^i, but is considerably smaller. The time for the entire bulk of the plasma to spin up to the speed of the plasma just outside the separatrix is then about 6×10^{-4} sec. Unless the plasma on the end shorted field line is rotating at least 10 times faster than the ion diamagnetic speed, classical viscosity does not seem able to generate the observed spin up in FRX-B. To summarize, the experimental evidence indicates that resistivity and shear viscosity (or equivalently, the off diagonal part of the stress tensor) are anomalously large, but that thermal conduction is not far from classical.

III. SIMILARITY SOLUTIONS FOR RESISTIVE DECAY

As was shown in the previous section, experiments on FRX-B indicate that the decay is roughly exponential in time, that the separatrix remains fixed in space, and that the density profile does not change shape as the plasma decays. All of this suggests that the equations for the resistive decay have similarity solutions. Furthermore, the extreme elongation of the plasma and the fact that the length does not change appreciably after the compression period, indicate that a one-dimensional model should suffice. We find that this is indeed the case for resistive decay of a reversed field configuration in slab geometry. In slab geometry, the equations for resistive diffusion are

$$\frac{\partial A}{\partial t} + v\frac{\partial A}{\partial x} = \frac{\eta c^2}{4\pi}\frac{\partial^2 A}{\partial x^2} \tag{1a}$$

$$\frac{1}{8\pi}\left(\frac{\partial A}{\partial x}\right)^2 + nT = n_o T_o \tag{1b}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} nv = 0 \quad . \tag{1c}$$

Here, $x = o$ is the position of the field null, and the separatrix is taken to be at $x = x_o$. The quantities $A = A_y$ and n have even symmetry in x, whereas $B = B_z$ and $v = v_x$ have odd symmetry. The quantities n_o and T_o are respectively the density and temperature at $x = o$, and they are functions of time. Because the symmetry, n is a function of magnetic flux.

Equations (1) must be supplemented by an equation for the temperature. Since we have seen that thermal conduction is not the dominant transport mechanism, we assume $T = T_o\left(\frac{n}{n_o}\right)^{\gamma-1}$, where γ is the specific heat ratio. We consider principally $\gamma = 1$ (isothermal) because in FRX-B experiments, the temperature seems to be uniform in space. Our discussion of Eqs. (1) assume $\gamma = 1$. However, generalization to other values of γ is trivial and we present also results for $\gamma = 5/3$.

To generate a similarity solution to Eqs. (1), let us assume $A(x,t) = A(x)e^{-st}$, $n(x,t) = n(x)e^{-2st}$ ($e^{-2st/\gamma}$ in the general case), $v(x,t) = v(x)$ and T constant in space and time (i.e., $T(x,t) = T_o$ for an isothermal plasma). The total magnetic flux is zero inside the separatrix. On the separatrix, there is some field $B(x=x_s) = \left.\frac{\partial A}{\partial x}\right|_{x=x_s}$, and for $x > x_s$ there is vacuum so that B(x) has the same value it has on the separatrix plus any jump required to contain the pressure on the separatrix. Hence our similarity solution assumes that the field in the vacuum decays at the same rate as the field in the plasma. That is, the boundary condition on the vacuum wall necessary for the existence of a similarity solution is that magnetic flux is just absorbed by the wall at a rate dictated by its decay in the plasma. That is, the vacuum wall is "black" to the vacuum flux. Specifically, if the vacuum wall conserves flux (i.e., is a perfect conductor), no similarity solution is possible if this wall is off the separatrix. However, a similarity solution exists if a perfectly conducting wall is at the separatrix.

To find the similarity solution and the eigenvalue s we have devised an iteration scheme. First we assume a density profile n(x) and an eigenvalue s and solve for v from Eq (1c);

$$v(x) = \frac{2s}{n(x)} \int_0^x dx' \, n(x'). \tag{2}$$

Then using this form for v, we solve (1a)

$$-sA + v(x) \frac{\partial A}{\partial x} = \frac{\eta c^2}{4\pi} \frac{\partial^2 A}{\partial x^2} \tag{3}$$

for A(x) and the eigenvalue s, imposing boundary conditions $A(x_s) = 0$ and $\left.\frac{\partial A}{\partial x}\right|_{x=0} = 0$. The vector potential must be equal to zero, rather than some constant, on the separatrix because the enclosed flux is zero for all time. Thus $E_y = -1/c \, \partial A_y/\partial t = 0$ on the separatrix. The only value of A_y consistent with both this and the assumed time dependence of the similarity solution is $A(x_s) = 0$.

To normalize A, choose a ratio of density at the center to density at the separatrix. This specifies $\frac{\partial A}{\partial x}$ just inside the separatrix from pressure balance as dictated by Eq. (1b). Thus $n(x=x_s)/n(x=o)$ is required as a boundary condition. The exact determination of this quantity undoubtedly involves the physics of the separatrix. For our purposes, we simply regard it as a parameter and examine the similarity

solution as a function of it. Then to continue, we solve the pressure balance, Eq. (1b) for $n(x)$. This completes one cycle of the iteration.

In practice, this iteration scheme converges very rapidly and a self similar solution is found. In our similarity solutions, there is no requirement that η is constant as a function of x. In fact an anomalous resistivity might well be larger near the separatrix where the gradients are steeper. To study this we have taken $\eta(x) = [(1-\alpha) + 2\alpha x/x_s]\bar{\eta}$. The average $\bar{\eta}$ is independent of α; $\alpha = 0$ corresponds to uniform resistivity and $\alpha = 1$ has η peaked at the edge. For $\alpha = 0.6$, profiles of $A(x)$ and $B(x)$ are shown in Figure 2a, while profiles of $n(x)$ and $v(x)$ are shown in Figure 2b, assuming $n(x=x_s)/n_o = 1/2$ $(B(x=x_s)/B_o=0.71)$. The distance is normalized in units of x_s and the velocity is normalized in units of $\eta c^2/4\pi r_s$. The eigenvalue s = 1.03 $\left(\frac{\eta c^2}{4\pi r_s^2}\right)$. Clearly the self similar solution predicts very reasonable profiles of density, field and velocity. Figure 3 shows the dependence of $s/(\eta c^2/4\pi x_s^2)$ as a function of α for both $\gamma = 1$ and $\gamma = 5/3$ for $n(x_s)/n_o = 1/2$. For $\alpha = 0$, $\gamma = 1$, i.e., for uniform resistivity and an isothermal plasma, we have $s/(\eta c^2/4\pi x_s^2) = 1.31$. The decay rate decreases monotonically with α, the amount of peaking of η. It is interesting to note that s approaches a finite value as $\alpha \rightarrow 0$, although $\eta(0) = 0$ seems to imply $s \rightarrow 0$ from (3). This happens because with $\eta(0)$ small, there is little magnetic diffusion near x = o and therefore the current $(c/4\pi)\partial B/\partial x$ becomes large in such a manner that $\eta(0)(\partial B/\partial x)_{x=o}$ remains finite. This effect has also been observed by Drake et al.[8]

Let us now digress briefly to compare our similarity solutions of the coupled flow and magnetic diffusion with two simplified models. First assume the magnetic diffusion is uncoupled to the flow, so that the time dependence is governed by

$$-sA = \frac{\eta c^2}{4\pi} \frac{\partial^2 A}{\partial x^2} \tag{4}$$

subject to the same boundary conditions at $x = o$ and $x = x_s$, and with constant η. Then $s = \frac{\pi^2}{4} \frac{\eta c^2}{4\pi x_s^2}$, which is roughly twice as large as the decay rate s for the coupled problem. Thus the plasma configuration decays slower than the pure magnetic configuration. The reason is that in either case, the magnetic field decays because of the inward diffusion of field. However for the plasma configuration, the fluid motion is outward, and the plasma, of course, attempts to take the field back out with it. The net result is a reduction in the rate of magnetic decay.

The other simple model is pure plasma motion across a specified, time independent magnetic field, as is usually appropriate for a low beta plasma. In this case, flow balances Ohmic dissipation and

$$v = \frac{\eta c^2}{4\pi} \frac{\partial^2 A}{\partial x2} \Bigg/ \frac{\partial A}{\partial x} \; . \tag{5}$$

Assuming B is proportional to x and A satisfies the same boundary conditions, then $v = \frac{\eta c^2}{4\pi x}$ so that v diverges at the field null and has just the opposite structure in x to that in Figure 2b. Thus to get reasonable results, the time dependence of the magnetic structure must be taken into account. Specifically, at the field null, Ohmic dissipation is balanced entirely by the induced electric field, and not at all by the flow.

We have also studied the dependence of the decay rate on $n(x_s)/n_0$. The decay rate turns out to be nearly independent of this parameter. For instance Figure 4 shows a plot of s versus $n(x_s)/n_0$ for $\gamma = 1$ for $\alpha = 0$ and 0.6. There is almost no change in s as this parameter is varied. This leads to the very interesting conclusion that the decay rate is very insensitive to the boundary condition on the separatrix.

To summarize, the experimental data of FRX-B seems to be consistent with a similarity solution to the equations. We have found similarity solutions in slab geometry which give very reasonable profiles and decay rates. We have concluded that the decay rate remains finite if the resistivity at the neutral line goes to zero. Another interesting result is that a knowledge of the exact boundary condition on the separatrix is not necessary for an accurate calculation of the decay rate of the equilibrium.

IV. ANOMALOUS ROTATION GENERATION

In this section we examine how a current driven instability which gives rise to anomalous resistivity can also generate rotation of the plasma itself. Our basic assumption is that the fluctuating fields generated by the instability constitute the only transport mechanism; that is we neglect collisions and classical transport. To calculate the rotation generation, we assume first the distribution function is given by $f + \delta f$ where δf is the fluctuating part which is in phase with the fluctuating potential. That is if

$$\phi(x) = \sum \phi(k)e^{ikx} + c.c. \quad , \tag{6}$$

then

$$\delta f(x) = \sum \delta f(k)e^{ikx} + c.c. \tag{7}$$

where there is a particular phase relation between $\delta f(k)$ and $\phi(k)$ but where the $\phi(k)$ for different k have random phases with respect to one another.

In order to generate ensemble average fluid equations from the Vlasov equation, there are now two types of ensemble averages which are necessary. First there is the usual integration over particle velocity u (the symbols u, v denote individual particle velocities and fluid velocities, respectively); second there is the average over wave phases. We denote the former by an angle bracket $\langle A\rangle = \int d^3ufA$,

the latter by a top line. In what follows we assume the magnetic field has only a z component and that all ensemble average quantities vary only with r. The double average u_θ moment of the ion Vlasov equation is

$$\frac{\partial}{\partial t}nv_{i\theta} + \underline{\nabla} \cdot \langle \underline{u}_i \underline{u}_i \rangle \Big|_\theta + \frac{ne}{Mc} v_r B_z - \frac{e}{M} \overline{\delta E_\theta \delta n} - \frac{e}{M} \overline{E}_\theta n = 0 \qquad (8)$$

and the corresponding average of the electron Vlasov equation is

$$- \frac{ne}{mc} v_r B_z + \frac{e}{m} \overline{\delta E_\theta \delta n} + \frac{e}{m} \overline{E}_\theta n = 0 \quad . \qquad (9)$$

In deriving Eqs. (8) and (9) we have made several assumptions. First we have assumed quasi-neuturality so $n_i = n_e$ and $\delta n_i = \delta n_e$. Second we have assumed $v_{ir} = v_{er} = v$. In the appendix, it is shown that the slight difference between v_{ir} and v_{er} necessary for setting up the radial electric field E_r does not make a significant contribution to the rotation. Third, we have neglected both electron inertia and any electron transport resulting from an off diagonal component of the electron pressure tensor. Since the last three terms of Eq. (8) are zero because of Eq. (9), the equation for rotation generation is simply

$$\frac{\partial}{\partial t}nv_{i\theta} = - \underline{\nabla} \cdot \langle \underline{u}_i \underline{u}_i \rangle \Big|_\theta = - \frac{1}{r} \frac{\partial}{\partial r} r \langle u_{ir} u_{i\theta} \rangle - \frac{\langle u_{ir} u_{i\theta} \rangle}{r} \qquad (10)$$

The problem now is to calculate $\langle u_{ir} u_{i\theta} \rangle$. The approach here is similar to that taken in the calculation of anomalous transport due

to drift instabilities.[9] One takes the u_{ir}^2 and $u_{i\theta}^2$ moments of the ion Vlasov equation and subtracts the two. The result is

$$\frac{2eB}{Mc}\langle u_r u_\theta\rangle - \frac{e}{M}\langle u_\theta \overline{\delta E_\theta \delta f}\rangle + \frac{e}{M}\langle u_r \overline{\delta E_r \delta f}\rangle$$

$$- \frac{e}{M}E_\theta n v_\theta + \frac{e}{M}E_r n v_r = \left[-\frac{\partial}{\partial t}\langle u_\theta^2 - u_r^2\rangle - \frac{1}{r}\frac{\partial}{\partial r} r\langle u_r(u^2 - u_r^2)\rangle\right]/2 \quad (11)$$

where we have dropped the i subscript. It can be shown that the right hand side of Eq. (11) gives rise to a contribution which is small by $(\omega_{ci}T)^{-1}$ or ρ_i/L where T and L are macroscopic time and length scales. We neglect this and set the left hand side of Eq. (11) to zero. Thus

$$\langle u_r u_\theta\rangle = \frac{c}{2B}\left[\langle u_\theta \overline{\delta E_\theta \delta f}\rangle - \langle u_r \overline{\delta E_r \delta f}\rangle + E_\theta n v_\theta - E_r n v_r\right] . \quad (12)$$

Inserting $\langle u_r u_\theta\rangle$ from Eq. (12) into Eq. (10) gives an equation for the generation of rotation from a fluctuating field. Note that the rotation generation is a bulk effect and occurs everywhere in the plasma. This is quite different from other models[5] which invoke rotation by end shorting outside the separatrix, and then spin up, via classical viscosity, inside. Furthermore, up to now, there has been no need to identify any particular instability. One attractive feature of this model is that the same instability responsible for anomalous resistivity could also be responsible for spin up. This seems to be a reasonable hypothesis, because the time to lose half the plasma is measured to be

roughly the same as the time for the plasma to spin up to the ion diamagnetic velocity.[1] We will see that by making only very weak assumptions concerning the micro-instability, it is possible to compare the diffusion time to the spin up time. It turns out that the plasma does indeed accelerate to roughly the ion diamagnetic velocity in a resistive diffusion time.

We now examine qualitatively the rotation of the plasma assuming that the fluctuating fields arise from a lower hybrid drift instability. Assume that initially there is no plasma rotation, so that the ions are confined electrostatically and

$$E_r = \frac{T_i}{ne}\frac{\partial n}{\partial r} \tag{13}$$

assuming T_i has no spatial variation. Then if $T_e << T_i$, the electrons rotate with drift velocity

$$V_{\theta e} = - c\,\frac{E_r}{B_z} \ . \tag{14}$$

Since $\frac{\partial n}{\partial r} < 0$ in the outer part of the plasma, $v_{e\theta} > 0$. According to Reference 10, the fastest growing electrostatic lower hybrid drift instability has $k_\theta \approx (2\ m/M)^{\frac{1}{2}}\omega_{ce}/v_i$, $k_r \approx 0$ and $\omega/k_\theta \approx v_{\theta e}/2$, where v_i is the ion thermal velocity. If $\delta E_r = 0$, the quantity to calculate is $\langle u_\theta \overline{\delta E_\theta \delta f}\rangle$. Since the frequency of the lower hybrid drift instability is much larger than the ion cyclotron frequency, the ions are effectively unmagnetized, so

$$\delta f = \frac{\frac{-e\delta E_\theta}{M}\frac{\partial f}{\partial u_\theta}}{-i(\omega + i\varepsilon - k_\theta u_\theta)} \tag{15}$$

where the sign of the $i\varepsilon$ is from the causality condition and it tells which way to integrate around the singularity. Making use of the fact that the drift velocity is much less than the ion thermal speed, we find

$$\langle u_\theta \overline{\delta E \delta f} \rangle = \frac{e|\delta E_\theta|^2}{M} \frac{n v_{e\theta}^2}{4|k_\theta| v_i^3} \sqrt{\frac{\pi}{2}}.$$

using the fact that $\omega/k_\theta = v_{e\theta}/2$.

Let us first calculate the volume average rate of change of angular momentum, and then compare the rate of spin up to the rate of resistive diffusion. Since the right hand side of Eq. (10) is the divergence of a tensor, we can do a volume integral for $r < r_s$ where r_s is the radius of the spearatrix, and find

$$\frac{d}{dt}\int d^3r\, n v_{i\theta} = - \langle u_r u_\theta \rangle \Big|_{r_s} 2\pi r_s L$$

$$= -\pi r_s L c \left[\frac{1}{B}\left(\frac{e|\delta E_\theta|^2}{M}\sqrt{\frac{\pi}{2}}\,\frac{n v_{e\theta}^2}{4\pi |k_\theta| v_i^3} - E_r n v_r\right)\right]_{r = r_s} \tag{16}$$

where L is the length of the plasma. The $E_\theta v_\theta$ term does not contribute to Eq. (16) because E_θ is zero at the separatrix (which is assumed fixed in time) because the enclosed magnetic flux is zero at all times, and the $\langle u_r \overline{\delta E_r \delta f} \rangle$ term is zero because k_r is assumed zero. Since $E_r \Big|_{r=r_s} < 0$ and $v_r \Big|_{r=r_s} > 0$, the right hand side of Eq. (16) is negative, so the plasma spins up in the negative theta direction, that is, in the diamagnetic direction.

It is interesting that the phase velocity of the instability is in the positive θ direction, so that the force exerted directly on the ions is also in the positive θ direction. However the plasma spins up in the opposite direction. The explanation, of course, is that the net force on the ions is not a force on the plasma; there is an equal and opposite force on the plasma electrons. This force between electrons and ions cannot rotate the plasma, but gives rise to anomalous resistivity and particle loss. The rotation results from an off diagonal contribution to the pressure tensor. It is analogous to classical viscosity, except that this off diagonal term does not arise from a velocity shear, but rather from an instability driven by the relative electron-ion drift $v_{e\theta}$.

It is useful now to see how much the plasma spins up in a resistive diffusion time. The resistivity can most easily be calculated in terms of the anomalous force density on the ions $F_\theta = e\,\overline{\delta n \delta E_\theta}$ and it is

$$\eta = \frac{F}{n^2e^2v_{e\theta}} = \frac{|\delta E_\theta|^2}{M k_\theta n}\sqrt{\frac{\pi}{2}}\,\frac{1}{2v_i^3} \ . \tag{17}$$

Then the resistive diffusion time is $\tau_r^{-1} = \frac{\eta c^2}{4\pi r_s^2}$ and the outward radial velocity is given roughly by $v_r \approx r_s/\tau_r$. With this, one can show that the two terms on the right hand side of Eq. (16) are about the same size assuming E_r is given by Eq. (13), and also that the plasma spins up to about the ion diamagnetic frequency in about a resistive diffusion time.

In calculating the ratio of spin up time to resistive diffusion time, it is interesting to note that this ratio is independent of both k_θ and $|\delta E_\theta|^2$. Thus the ratio depends only on the phase velocity of the fluctuation, which we assume is $v_{e\theta}/2$. This is a very weak assumption, because no matter what the instability is, or what saturates it, the phase velocity will almost certainly be between the ion and electron rotation speeds.

To summarize, we have shown that in FRX-B, the anomalous resistivity and spin up might both result from the same cause, namely an instability driven by the θ current. The fact that the plasma is observed to spin up to the ion diamagnetic frequency in about a resistive diffusion time lends credence to this hypothesis. By making only very weak assumptions concerning the instability (taken to be a lower hybrid drift instability in our case) it is possible to show that the plasma spin up is in the right direction and that it accelerates to about the ion diamagnetic velocity in about a resistive diffusion time.

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We wish to thank R. K. Linford and J. F. Drake for stimulating discussions.

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APPENDIX

In this appendix we show that the difference in radial diffusion velocity of electrons and ions induce only a negligible amount of plasma rotation. To do so, consider the ion and electron momentum equations in the θ direction neglecting all off diagonal terms in the pressure tensor. They are

$$\frac{\partial}{\partial t} n_i M v_{i\theta} + \frac{1}{r}\frac{\partial}{\partial r} r n M v_{ir} v_{i\theta} + \frac{n M v_{ir} v_{i\theta}}{r} = n_i e E_\theta - n_i e \frac{v_{ir}}{c} B - F_{ei\theta} \quad \text{(A-1)}$$

$$0 = -E_\theta + \frac{v_{er}}{c} B + \frac{F_{ei\theta}}{n_e e} \quad \text{(A-2)}$$

where $F_{ei\theta}$ is the force density action from one species on another. Also we now explicitly account for small differences in electron and ion densities and radial velocities. Substituting for E_θ and neglecting the F term, which is small both because F (the collisionality) is small and because it is multiplied by $n_e - n_i$, we find that the equation for rotation generation is

$$\frac{\partial}{\partial t} n M v_{i\theta} + \frac{1}{r}\frac{\partial}{\partial r} r n M v_{ir} v_{i\theta} + \frac{n M v_{ir} v_{i\theta}}{r} = \frac{neB}{c} w \quad \text{(A-3)}$$

where we adopt the notation $w = v_{er} - v_{ir}$. This equation relates $v_{i\theta}$ to w. There are two other equations, Maxwell's equation relating E_r to w

$$\frac{\partial E_r}{\partial t} = 4\pi ne\ W \tag{A-4}$$

and the radial ion mementum equation

$$E_r = -\frac{v_{i\theta}}{c}\ B + \frac{1}{ne}\frac{\partial p_i}{\partial r} \tag{A-5}$$

where we have neglected the centrifugal force in Eq. (A-5). Thus Eqs. (A-3)-(A-5) relate the quantities w, $v_{i\theta}$ and E_r to each other and to the other fluid quantities. However if the centrifugal force, which is small compared to the pressure gradient is not included in the pressure balance equation, the equations for the other fluid quantities n, v_r, T_e, T_i and B do not couple to $v_{i\theta}$, w and E_r. Thus Eqs. (A-3)-(A-5) are equations for the rotation which can be solved after the other fluid quantities are known. We now show that the rotation generated in this way is negligible.

The idea is that a change in ion pressure p_i in time due to the decay of the plasma produces a displacement current $\frac{\partial E}{\partial t}$ according to Eq. (A-5). This in turn produces a current-new according to Eq. (A-4), which generates a rotation, according to Eq. (A-3). A simple calculation shows that the rotation generated in the decay time is of order $(\Omega_{ci}/\omega_{pi})^2 v_{Di}$, which for a dense plasma like FRX-B is much less than the observed rotation velocity, of order v_{Di}. Thus ordinary resistive transport can explain the decay of the plasma, but not its spin up. To explain the spin up, an off diagonal contribution to the pressure tensor is required. This can come from either a classical transport effect, i.e., shear viscosity, or anomalous transport, i.e., a current generated instability.

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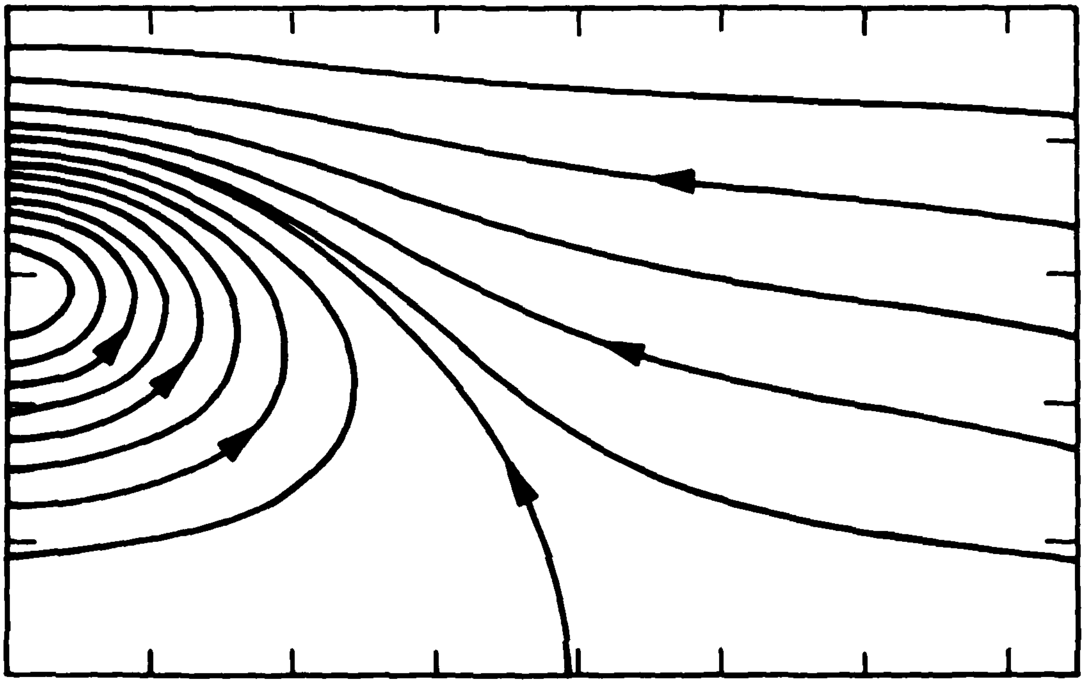

Fig. 1 — Flux surfaces for a typical RFP equilibrium. The scale is compressed in the axial (z) direction.

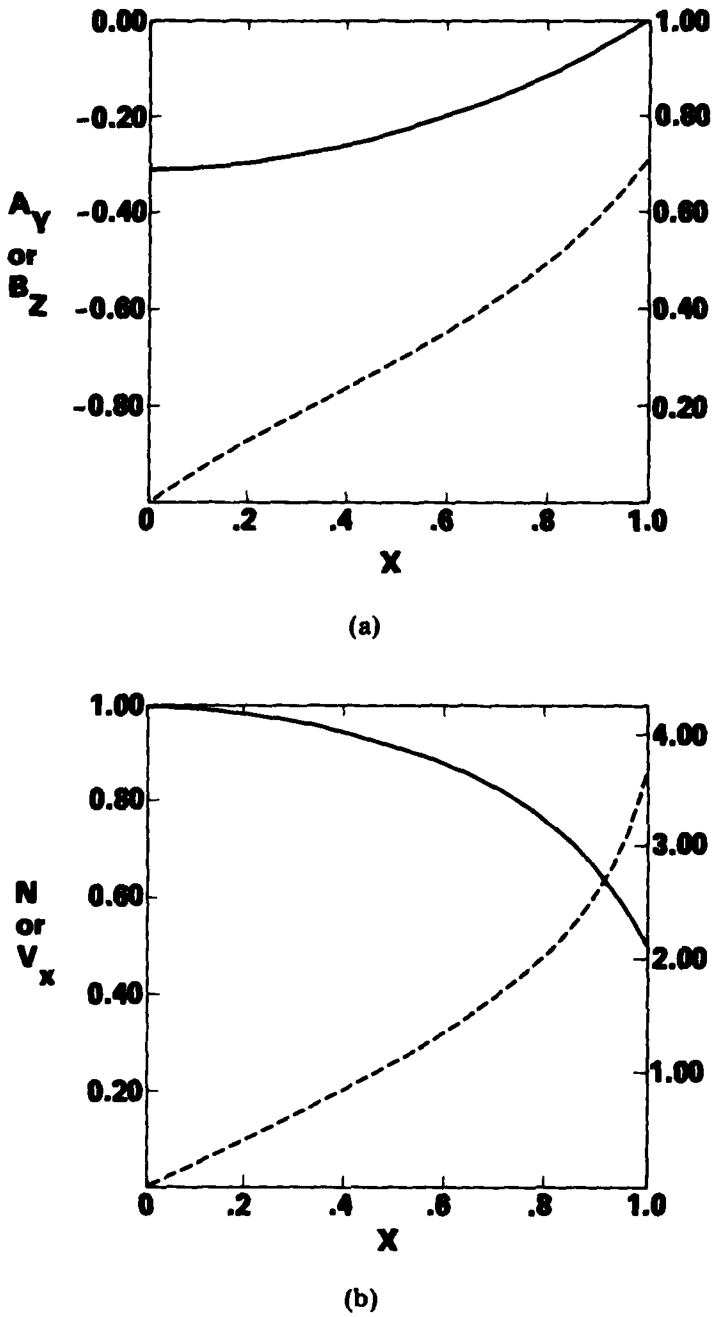

Fig. 2 — Profiles of (a) vector potential A_y (solid) and magnetic field B_z (dotted), (b) density (solid) and velocity v_x (dotted) as functions of the normalized distance from the separatrix for $\alpha = 0.6$ and $n(x_s)/n_o = 1/2$

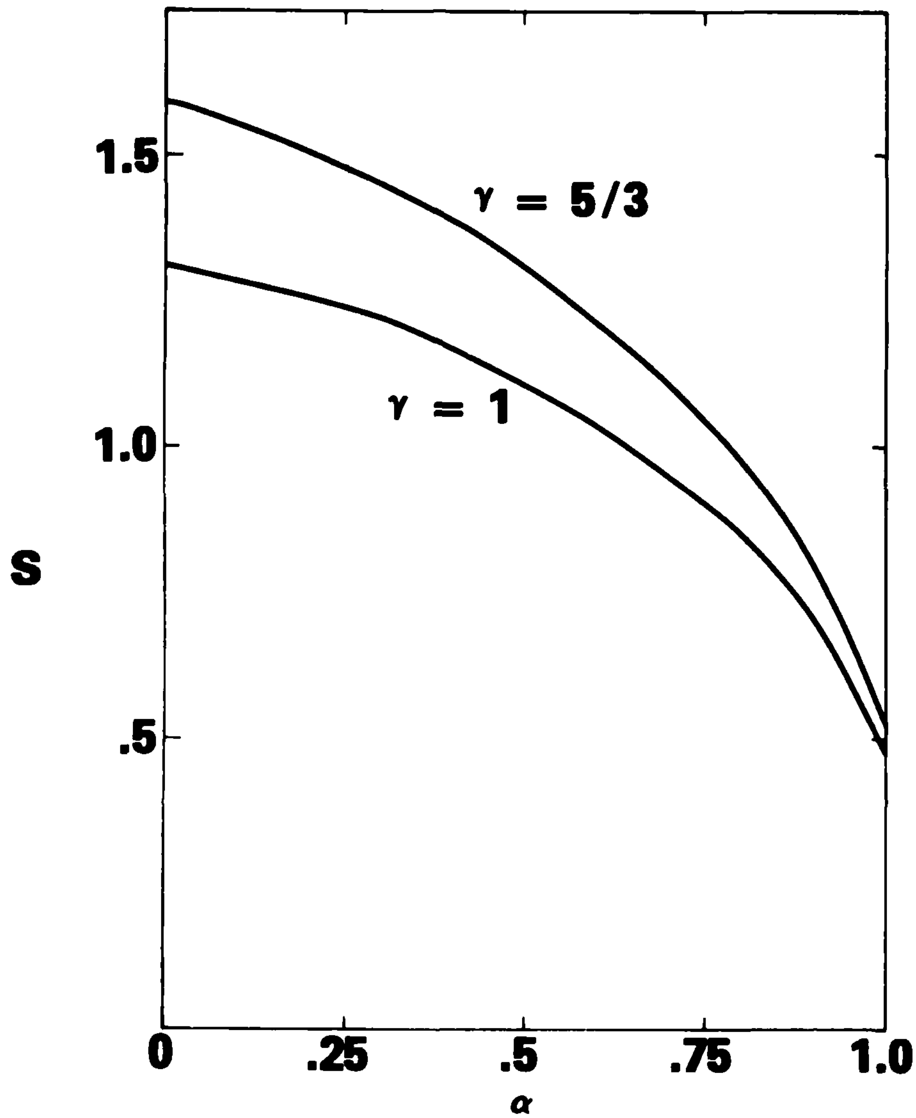

Fig. 3 — Normalized decay rate $S = s/(\bar{\eta}c^2/4\pi x_s^2)$ as a function of the peaking parameter α for $\gamma = 1$ and $\gamma = 5/3$, and for $n(x_s)/n_o = 1/2$. Notice that S remains finite as α approaches zero, i.e., $\eta(o) \to o$.

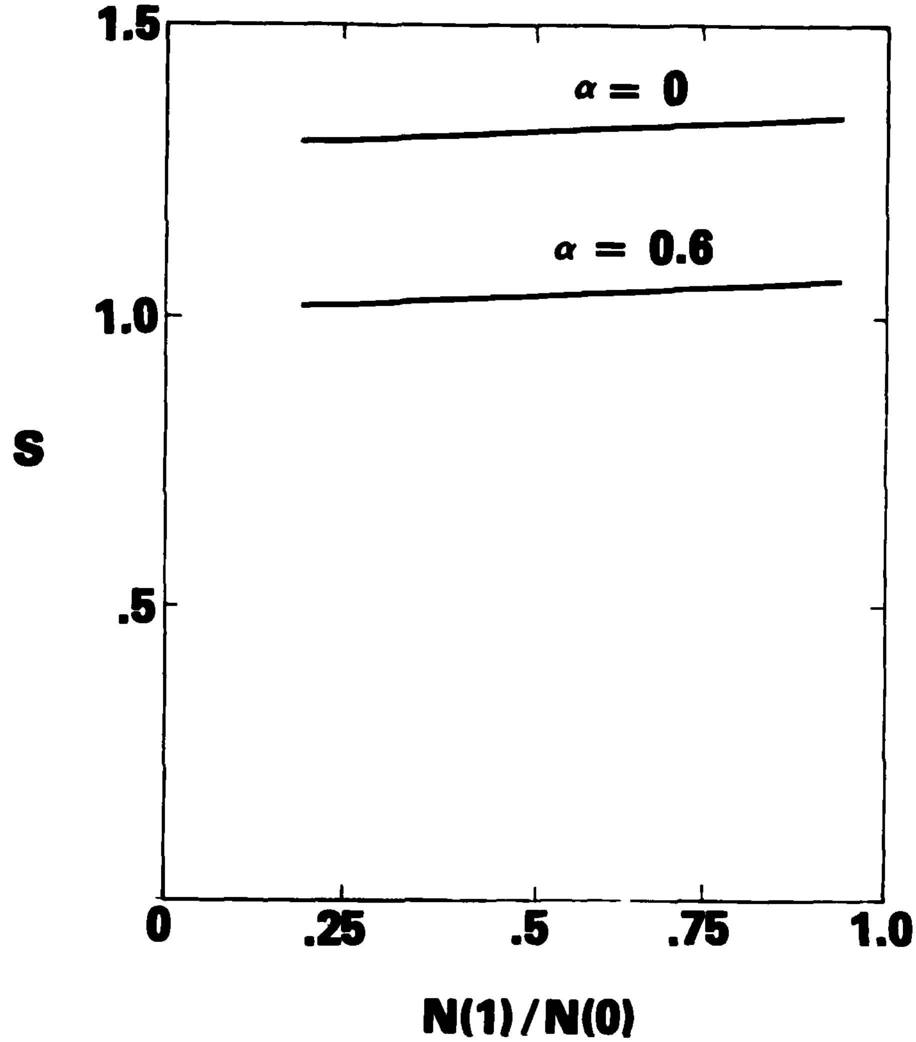

Fig. 4 — Normalized decay rate $S = s/(\bar{\eta}c^2/4\pi x_s^2)$ as a function of the normalized separatrix density $n(x_s)/n_o = N(1)/N(0)$ for $\gamma = 1$ and $\alpha = o$ and 0.6. Note that S is nearly independent of the density (and hence pressure) on the separatrix.

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